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**Authorship Statement**

**This dissertation is based on the results of research carried out by myself, is my own composition, and has not been previously presented for any other certified or uncertified qualification**

**This research was carried out under the supervision of <Supervisor Name>**

**DATE – SIGNATURE**

**Copyright Statement**

**In submitting this dissertation to the…**

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**ACKNOWLEDGEMENTS**

**I would like to express my sincere gratitude…**

**ABTSTRACT**

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**List Of Abbreviations**

**P** Polynomial Time

**NP** Non-deterministic Polynomial Time

**TSP**  Travelling Salesman Problem

**SAT** Boolean Satisfiability

**UNSAT**  Unsatisfiable

**CNF** Conjunctive Normal Form

**DPLL** Davis-Putnam-Logemann-Loveland

**CDCL** Conflict-driven clause learning

**UIP** Unique Implication Point

**VSIDS** Variable State Independent Decaying Sum

**Theoretical Background**

**2.1 P vs NP**

The P vs NP problem is one of the seven “Millenium Problems”, which are a set of famous unsolved problems in mathematics, founded by Landon T. Clay in 1998 [1]. In principle, the P vs NP problem asks whether every problem whose solution can be quickly verified can also be quickly solved.

**2.1.1 Polynomial Time**

P(Polynomial Time) refers to the class of decision problems that a deterministic algorithm can solve in polynomial time – meaning that the time required to solve the problem grows at most as some fixed power of the input size [2].

A graph of different colored arrows

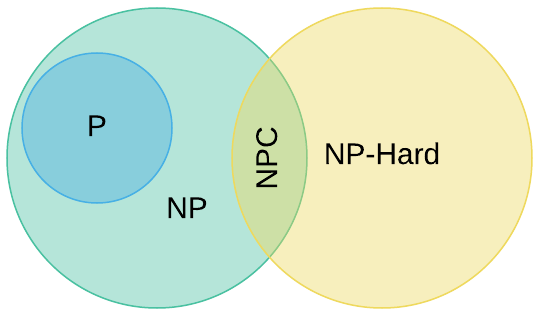
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**Figure 2.1**: Time Complexity [3]

Figure 1 illustrates common algorithmic time complexities. While quadratic time, O(n²), is relatively inefficient compared to linear or logarithmic time, it still falls within the class of polynomial-time algorithms. In contrast, exponential time, O(2ⁿ), and factorial time, O(n!), grow much faster and lie outside the class P

**2.1.2 Non-deterministic Polynomial Time**

NP(Nondeterministic Polynomial Time) refers to a problem for which a given solution can be verified in polynomial time by a deterministic algorithm – even if finding that solution could prove difficult [3].



**Figure 2.2**: Complexity Classes [4]

Figure 2 shows the complexity classes working together, it clearly depicts P ⊆ NP always. The table below describes the key relationships between the class and its function (NP-Complete and NP-Hard are followed through in sections 2.3 and 2.4 respectively):

| **Class** | **Description** |
| --- | --- |
| **P** | Problems solvable quickly (polynomial time) by deterministic algorithms. |
| **NP** | Problems whose solutions can be verified quickly, includes all of P. |
| **NP-Complete** | The hardest problems within NP. Efficiently solving one implies efficient solutions for all NP problems. |
| **NP-Hard** | Problems at least as hard as NP-complete, but not necessarily in NP-or even decidable. |

**Table 2.1**: Complexity Class Functions

**2.1.3 NP-Complete**

NP-complete problems are a subset of NP with two defining properties:

* **In NP** - Their solutions can be verified in polynomial time by a deterministic algorithm.
* **NP-hardness** - they are at least as hard as any problem in NP, meaning every NP problem can be transformed (reduced) to them in polynomial time [5].

Such example of an NP-Complete problem is the Traveling Salesman Problem (TSP), given a list of cities and the distances between them, the task is to determine the shortest possible route that visits each city exactly once and returns to the starting point. Verifying a proposed route’s total distance is fast (polynomial time), but finding the optimal route may require checking exponentially many possibilities - making TSP a classic NP-complete problem [5]

**2.1.4 NP-Hard**

NP-hard problems are those to which every problem in NP can be reduced in polynomial time, making them at least as challenging as NP-complete problems. Importantly, NP-hard problems are not required to be in NP - they may lack efficient verifiers or even be undecidable. In contrast, NP-complete problems must be both in NP (verifiable in polynomial time) and NP-hard, placing them among the most difficult problems within NP [6].

Such example of an NP-Hard problem is the Halting Problem, which asks whether a given computer program will eventually stop running (halt) or continue executing forever, for a specified input. Alan Turing proved in 1936 that there is no general algorithm capable of solving this problem for all possible program–input pairs, making it undecidable [7].

**2.1.5 P vs NP Conclusion**

Hence, P = NP is equivalent to stating “Is every problem whose solution is easy to check, also easy to solve?”. This question is one of the most important open problems in theoretical computer science, with far-reaching implications. If P = NP were proven true, a wide range of problems currently thought to be intractable could be solved efficiently, revolutionizing fields such as cryptography, optimization, logistics, artificial intelligence and bioinformatics amongst others. Conversely, if P != NP, it would confirm that certain problems are incapable of efficient solutions.

**2.2 Boolean Satisfiability (SAT)**

Boolean Satisfiability is the task of determining whether a Boolean formula can be made true by assigning truth values to its variables. Given a propositional formula: ϕ(x₁, x₂, …, xₙ), SAT checks whether there exists an assignment for the given literals such that ϕ evaluates to True [8]. Example:

(x ∨ y) ∧ (¬x V z) is satisfiable with the assignment:

X = False, y = True, z = True

**2.2.1 Conjunctive Normal Form (CNF)**

To apply SAT solvers, which will be discussed on later throughout the paper – Boolean formulas are expressed in Conjunctive Normal Form:

* Literal: A variable (x) or its negation (¬x)
* Clause: A disjunction (OR) of literals, e.g., (x V ¬y V z)
* CNF Formula: A conjunction (AND) of Clauses, e.g.:

(x V y) ∧ (¬x V z) ∧ (¬y V ¬z)

This representation is central due to most SAT algorithms operating on CNF [9].

**2.2.2 3-SAT**

3-SAT is a restricted version of SAT where each clause contains at most three literals [10] . Example:

(x V ¬y V z) ∧ (¬x V y V w)

3-Sat is highly significant as it was one of the first problems shown to be NP-Complete similarly to SAT, in contrast to 2-SAT, which is solvable in polynomial time [10]. Due to its central role in complexity theory and its use as a benchmark for satisfiability algorithms, 3-SAT has become a primary focus of research - and it will likewise be the focus of this thesis.

**2.2.3 SAT Conclusion**

As the first problem proven to be NP-Complete, SAT lies at the heart of the P vs NP question. Its significance reaches beyond theory since it is universal – due to a wide range of problems such as graph coloring, scheduling and circuit verification can be efficiently reduced to SAT.

Moreover, SAT’s practical relevance in modern SAT solvers are capable of handling instances with millions of variables – which will be discussed later in the thesis. As a result, SAT-based approaches have become indispensable tools in diverse applications including software and hardware verification, security analysis, artificial intelligence and cryptography.

Together, these properties make SAT not only a pivotal theoretical construct but also a practical framework for solving real-world problems. Thus, SAT and the P vs NP problem are inseparably connected which explains why SAT continues to attract significant research interest and why it serves as a natural focus for this thesis.

**Literature Review**

**3.1 SAT Solvers**

A SAT solver is a tool that takes a CNF Formula as input and outputs either a satisfying Boolean assignment or an UNSAT if it is not [11]. SAT solvers provide combinatorial reasoning with the underlying representational formalism being propositional logic. However, the full potential of SAT solvers becomes apparent in their practical uses that are not viewed as propositional reasoning tasks such as [12]:

* AI Planning
* Hardware Verification
* Cryptanalysis
* Scheduling

Multiple SAT solvers have been created throughout the years, with the first instance being the Davis-Putnam-Logemann-Loveland algorithm created in the 1960s [11]. Afterwards, the Conflict-driven clause learning algorithm was created during the mid-90s as an improvement on the DPLL algorithm [13].

Furthermore, more modern and robust SAT solvers have been created throughout the years

**3.2 DPLL Algorithm**

The Davis-Putnam-Logemann-Loveland algorithm underlies most modern SAT solvers. It was introduced by Martin Davis, Hilary Putnam, George Logemann and Donald Loveland. However, it had improvements performed over the 1960s by Davis-Putnam [14]. The DPLL algorithm works as such:

1. **Unit Propagation**

* If a clause has only one literal, that literal must be true to satisfy the clause.
* Assign it as true, remove all clauses containing it, and delete its negation from other clauses.

1. **Pure Literal Elimination**

* If a variable appears with only one polarity (always positive or always negated), assign it to satisfy all clauses containing it.
* Remove the satisfied clauses from the formula.

1. **Splitting / Decision Step**

* Select an unassigned variable and assign it a truth value (e.g., true).
* Recursively check satisfiability under this assignment.
* If this leads to a contradiction (UNSAT), backtrack and try the opposite assignment (false) as shown below.

1. **Backtracking**

* When a conflict is detected, backtrack to the most recent decision point and try an alternative assignment.
* If no alternatives remain, the formula is unsatisfiable [15].

The below depiction shows pseudocode of the DPLL algorithm

A screenshot of a computer program

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**Figure 3.1**: DPLL-Recursive(F, ρ) [12]

DPLL is inefficient due to its limitations:

* It does not keep track of what it learns – With each backtrack, it does not track why a conflict occurred.
* It may revisit the same conflict pattern multiple times
* Even though it backtracks, it only undoes the most recent decision even If the real cause of the conflict may of have occurred several levels deeper. [16]

Furthermore, a raw full coding solution of the DPLL algorithm can be found in Appendix A via the help of an LLM, no external packages were used.

**3.3 CDCL Algorithm**

The Conflict-Driven Clause Learning Algorithm is an extension over DPLL which is the main algorithmic paradigm used in modern SAT solvers today, retaining many components such as unit propagation and backtracking. However, it adds several powerful add-ons which are particularly effective on large, real-world formulas [16]. The CDCL algorithm works as follows(steps are listed in approximate order, though many work concurrently):

**1. Unit Propagation (slightly different to DPLL)**

* Unit propagation consists of Decision and Implication assignment types – where decisions are randomly assigned using heuristics and implications are forced assignments due to the state of the current clause
* Tracks why every propagated literal was assigned, this is later used in conflict analysis and non-chronological backtracking

**2. Conflict Detection**

* If unit propagation produces a clause (typically at implication level) where all literals are false, a conflict occurs
* Unlike DPLL, CDCL does not simply backtrack chronologically to the last decision. Instead, it analyses the conflict to avoid repeating the same mistake as depicted below

**3. Conflict Analysis (Clause Learning)**

* For conflict analysis, an implication graph is created

An implication graph is a directed graph that depicts how propagations were made by the engine [18]. In this case  
A diagram of a number flow

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**Figure 3.2**: Implication graph example [18]

* Orange boxes represent decision nodes, which correspond to the variables that the engine assigns during the decision process.
* Dark green boxes represent implication nodes, whose truth values are determined automatically through unit propagation due to earlier decisions.
* The light green box represents the conflict node, indicating that the current assignment violates at least one clause of the formula.
* The @ symbol indicates the decision level at which each assignment was made [18].

The process is as follows:

* 1. The solver selects a variable and assigns it a truth value, creating a decision node (orange box).
  2. This assignment triggers unit propagation, leading to additional implied assignments represented by implication nodes (dark green boxes).
  3. Steps one and two are repeated across successive decision levels, building a chain of implications.
  4. Eventually, a conflict arises when a clause becomes unsatisfied under the current assignments, represented by the conflict node (light green box).
  5. The solver then performs backtracking (backjumping) to a previous decision level to revise the assignment and continue the search, this is further explained in step 4 [18].
* An important part of conflict analysis is the Unique Implication Point (UIP) which represents the point in the implication graph where all propagation paths from the most recent decision literal converge together before a conflict. In figure 3.2, our UIP is ‘*x70 @56*’ since it is the first node at the current decision level through which all implication paths from the latest decision literal (−x73 @ 56) pass before reaching the conflict. This point marks where conflict analysis stops and the learned clause is derived, preventing the solver from repeating the same conflicting assignment in future search steps [18].

**4. Non-Chronological Backtracking (Backjumping)**

* After learning a clause, CDCL determines the earliest decision level where the conflict could have been avoided.

Instead of undoing just the last decision:

* CDCL jumps back directly to that decision level.
* It then reapplies unit propagation with the learned clause.

The learned clause indicates the earliest decision level where the conflict could have been prevented. Instead of merely undoing the last decision (chronological backtracking), the solver jumps directly to this determined level [19].

**5. Decision Heuristics**

The goal of this step is to make a “good” guess that will lead to a quick solution or a conflict that yields a new learned clause. Two popular heuristics are:

* + Variable State Independent Decaying Sum (VSIDS)
  + Phase Saving

VSIDS is a widely used heuristic that focuses on the history of recent conflicts. It works as follows:

* 1. Every literal has an associated score, initially zero
  2. Once a conflict occurs, the score of each literal that belonged to the newly learned clause is incremented by a fixed amount (usually one)
  3. Periodically, the scores of all literals are divided by a constant factor (e.g. two). This causes the influence of older conflicts to decay over time, prioritizing the variables involved in newer conflicts.
  4. The heuristic selects the unassigned variable that has the highest score (sums the scores of both positive and negative literals for that variable) [20]

Phase Saving is another heuristic that determines which truth value (polarity) to assign to a chosen variable. It works as follows:

* + 1. The solver keeps track of the last successful polarity assigned to every variable before it was reset by backtracking
    2. Once a variable selection heuristic (such as VSIDS, but not necessarily) chooses a variable, the phase saving heuristic assigns that variable the same polarity it had the last time it was assigned [21].

These are both powerful decision heuristics as they ensure the solver quickly re-engages with the more complex parts of the formula and reuse the successful assignments from previous search branches.

**6. Restart Policy**

A restart policy is a heuristic that periodically discards the current assignment and starts the search from the beginning (Decision level 0). It is essential due to:

* + Needing to escape suboptimal search regions, since runtime for some instances can follow a heavy-tailed distribution. This means that the engine might occasionally get stuck in a long branch of the search tree, hence a restart gives the solver a “second chance” as a much shorter runtime.
  + Leveraging learned clauses, which are derived from past conflicts and remain in the databases.
  + Decision heuristics scores are kept, ensuring that the VSIDS will choose the same high-activity variables early on [22].
  + In past literature, restarts have shown to correlate with higher-quality clauses which are typically measured in Literal Block Distance(LBD). LBD is a metric used in CDCL solvers to measure the quality and usefulness of a learned clause. Clauses with low LBD are considered high-quality [23], it is calculated via function:

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**Figure 3.3**: Literal Block Distance Function [23]

**7. Clause Database Management**

This part of the algorithm periodically evaluates learned clauses and removes low-quality or redundant clauses. It is a necessary since as the solver runs, it learns a significant number of new clauses from conflicts – although these clauses guide the search and ensure completeness, their increasing volume creates a significant performance overhead. This makes use of the previously mentioned LBD [20].

The below depiction shows pseudocode of the CDCL algorithm

A screenshot of a computer code

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**Figure 3.2**: CDCL Algorithm Pseudocode [16]

Furthermore, a full coding solution of the CDCL algorithm can be found in Appendix B via the help of an LLM, the Pysat external package was used due to the sheer complexity of the CDCL algorithm. The glucose3 solver was used which is the CDCL algorithm with optimizations done to the clause database management component.

4.AND THEN FIND OUT WHAT ML SOLVERS EXIST AND DECIDE WHAT TO DO THESIS ON.

**Expressions Explainability in Appendix?**

P ⊆ NP – Every problem in P is automatically in NP.

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**Appendix A – DPLL Algorithm Implementation**

A screen shot of a computer program

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**Figure [Placeholder]**: DPLL Algorithm Implementation, part 1

A computer screen shot of a program

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**Figure [Placeholder]**: DPLL Algorithm Implementation, part 2

A screen shot of a computer program

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**Figure [Placeholder]**: DPLL Algorithm Implementation, part 3

**Appendix B – CDCL Algorithm Implementation**A screen shot of a computer program

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**Figure [Placeholder]**: CDCL Algorithm Implementation, part 1

A screen shot of a computer

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**Figure [Placeholder]**: CDCL Algorithm Implementation, part 2